**B-Math-III Mid Term Exam ; Differential Equations.**

Time : 3.00 hrs; Max Mark: 70 ; 9 March 2022

1. Obtain the differential equation for the family of curves given by $x^2+y^2-2cx=0$, where $c$ is a parameter. Hence find the differential equation for the orthogonal family of curves and solve it.(9+10)

2. The general solution $x(t)$ of the equation for a harmonic oscillator viz. $mx''+px'+qx=F_0\cos(\omega t)$ with underdamped $((\frac{p}{m})^2<\frac{4q}{m})$ and forced oscillations converges to a 'steady state' solution $x_\infty(t)$ as $t\to\infty$. Find $x_\infty(t)$ and explain why it is called a steady state solution. (8+8)

3. a) Show that $y(x):=v_1(x)y_1(x)+v_2(x)y_2(x), x\in[a,b]$ is a particular solution of the equation $y''+P(x)y'+Q(x)y=R(x)$ where $y_i, i=1,2$ are linearly independent solutions of the homogenous equation and the $v_i(x)$ are suitable indefinite integrals of $-\frac{y_i(x)R(x)}{W(x)}$. Here $P,Q$ and $R$ are continuous functions on $[a,b]$ and $W(x)$ is the Wronskian. (12)
b) Find the general solution of the equation $y''+y=\csc(x), x\in[\frac{\pi}{4},\frac{\pi}{2}]$. (7)

4. Let $y(x), x>0$ be a solution of the differential equation $x^2y''+xy'+(x^2-1)y=0$ and $u(x), x>0$ be a solution of $u''+(1-\frac{3}{4x^2})u=0$. Show that $y(x)=C\frac{u(x)}{\sqrt{x}}$ for some constant $C>0$. Hence deduce that $y(x)$ has atmost one zero in the interval $(0,\frac{\pi}{4})$. (8+6)

5. Obtain two linearly independent solution of Legendre's equation $(1-x^2)y''-2xy'+p(p+1)y=0$ in the interval $(-1,1)$. (12)